Project 3

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**Problem 1:** Transshipment Model

Part A

i.

min(15\*P1W1R1 + 16\*P1W1R2 + 17\*P1W1R3 + 20\*P1W1R4 + 27\*P1W2R3 + 23\*P1W2R4 + 25\*P1W2R5 + 29\*P1W2R6 + 16\*P2W1R1 + 17\*P2W1R2 + 18\*P2W1R3 + 21\*P2W1R4 + 20\*P2W2R3 + 16\*P2W2R4 + 18\*P2W2R5 + 22\*P2W2R6 + 18\*P3W1R1 + 19\*P3W1R2 + 20\*P3W1R3 + 23\*P3W1R4 + 20\*P3W2R3 + 16\*P3W2R4 + 18\*P3W2R5 + 22\*P3W2R6 + 23\*P3W3R4 + 21\*P3W3R5 + 21\*P3W3R6 + 15\*P3W3R7 + 26\*P4W2R3 + 22\*P4W2R4 + 24\*P4W2R5 + 28\*P4W2R6 + 22\*P4W3R4 + 20\*P4W3R5 + 20\*P4W3R6 + 14\*P4W3R7)

( Supply Constraints)

P1W1R1 + P1W1R2 + P1W1R3 + P1W1R4 + P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 <= 150

P2W1R1 + P2W1R2 + P2W1R3 + P2W1R4 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 <= 450

P3W1R1 + P3W1R2 + P3W1R3 + P3W1R4 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P3W3R4 + P3W3R5 + P3W3R6 + P3W3R7 <= 250

P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 + P4W3R4 + P4W3R5 + P4W3R6 + P4W3R7 <= 150

( Demand Constraints)

P1W1R1 + P2W1R1 + P3W1R1 >= 100

P1W1R2 + P2W1R2 + P3W1R2 >= 150

P1W1R3 + P1W2R3 + P2W1R3 + P2W2R3 + P3W1R3 + P3W2R3 + P4W2R3 >= 100

P1W1R4 + P1W2R4 + P2W1R4 + P2W2R4 + P3W1R4 + P3W2R4 + P3W3R4 + P4W2R4 + P4W3R4 >= 200

P1W2R5 + P2W2R5 + P3W2R5 + P3W3R5 + P4W2R5 + P4W3R5 >= 200

P1W2R6 + P2W2R6 + P3W2R6 + P3W3R6 + P4W2R6 + P4W3R6 >= 150

P3W3R7 + P4W3R7 >= 100

ii.

For this part we used Lindo. The input was as follows:

MIN 15 P1W1R1 + 16 P1W1R2 + 17 P1W1R3 + 20 P1W1R4 + 27 P1W2R3 + 23 P1W2R4 + 25 P1W2R5 + 29 P1W2R6 + 16 P2W1R1 + 17 P2W1R2 + 18 P2W1R3 + 21 P2W1R4 + 20 P2W2R3 + 16 P2W2R4 + 18 P2W2R5 + 22 P2W2R6 + 18 P3W1R1 + 19 P3W1R2 + 20 P3W1R3 + 23 P3W1R4 + 20 P3W2R3 + 16 P3W2R4 + 18 P3W2R5 + 22 P3W2R6 + 23 P3W3R4 + 21 P3W3R5 + 21 P3W3R6 + 15 P3W3R7 + 26 P4W2R3 + 22 P4W2R4 + 24 P4W2R5 + 28 P4W2R6 + 22 P4W3R4 + 20 P4W3R5 + 20 P4W3R6 + 14 P4W3R7

ST

! Supply Constraints

P1W1R1 + P1W1R2 + P1W1R3 + P1W1R4 + P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 <= 150

P2W1R1 + P2W1R2 + P2W1R3 + P2W1R4 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 <= 450

P3W1R1 + P3W1R2 + P3W1R3 + P3W1R4 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P3W3R4 + P3W3R5 + P3W3R6 + P3W3R7 <= 250

P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 + P4W3R4 + P4W3R5 + P4W3R6 + P4W3R7 <= 150

! Demand Constraints

P1W1R1 + P2W1R1 + P3W1R1 >= 100

P1W1R2 + P2W1R2 + P3W1R2 >= 150

P1W1R3 + P1W2R3 + P2W1R3 + P2W2R3 + P3W1R3 + P3W2R3 + P4W2R3 >= 100

P1W1R4 + P1W2R4 + P2W1R4 + P2W2R4 + P3W1R4 + P3W2R4 + P3W3R4 + P4W2R4 + P4W3R4 >= 200

P1W2R5 + P2W2R5 + P3W2R5 + P3W3R5 + P4W2R5 + P4W3R5 >= 200

P1W2R6 + P2W2R6 + P3W2R6 + P3W3R6 + P4W2R6 + P4W3R6 >= 150

P3W3R7 + P4W3R7 >= 100

END

And the results were:

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 17100.00

VARIABLE VALUE REDUCED COST

P1W1R1 0.000000 210.000000

P1W1R2 50.000000 0.000000

P1W1R3 100.000000 0.000000

P1W1R4 0.000000 5.000000

P1W2R3 0.000000 10.000000

P1W2R4 0.000000 8.000000

P1W2R5 0.000000 8.000000

P1W2R6 0.000000 9.000000

P2W1R1 100.000000 0.000000

P2W1R2 100.000000 0.000000

P2W1R3 0.000000 0.000000

P2W1R4 0.000000 5.000000

P2W2R3 0.000000 2.000000

P2W2R4 50.000000 0.000000

P2W2R5 200.000000 0.000000

P2W2R6 0.000000 1.000000

P3W1R1 0.000000 2.000000

P3W1R2 0.000000 2.000000

P3W1R3 0.000000 2.000000

P3W1R4 0.000000 7.000000

P3W2R3 0.000000 2.000000

P3W2R4 150.000000 0.000000

P3W2R5 0.000000 0.000000

P3W2R6 0.000000 1.000000

P3W3R4 0.000000 7.000000

P3W3R5 0.000000 3.000000

P3W3R6 100.000000 0.000000

P3W3R7 0.000000 0.000000

P4W2R3 0.000000 9.000000

P4W2R4 0.000000 7.000000

P4W2R5 0.000000 7.000000

P4W2R6 0.000000 8.000000

P4W3R4 0.000000 7.000000

P4W3R5 0.000000 3.000000

P4W3R6 50.000000 0.000000

P4W3R7 100.000000 0.000000

iii.

According to Lindo, you would want to do the following:

P1W1R2 50.000000 (ship 50 refrigerators from P1 to R1 through W1)

P1W1R3 100.000000 (ship 100 refrigerators from P1 to R3 through W1)

P2W1R1 100.000000 (ship 100 refrigerators from P2 to R1 through W1)

P2W1R2 100.000000 (ship 100 refrigerators from P2 to R2 through W1)

P2W2R4 50.000000 (ship 50 refrigerators from P2 to R4 through W2)

P2W2R5 200.000000 (ship 200 refrigerators from P2 to R5 through W2)

P3W2R4 150.000000 (ship 150 refrigerators from P3 to R4 through W2)

P3W3R6 100.000000 (ship 100 refrigerators from P3 to R6 through W3)

P4W3R6 50.000000 (ship 50 refrigerators from P4 to R6 through W3)

P4W3R7 100.000000 (ship 100 refrigerators from P4 to R7 through W3)

Part B

We can see from the cost table that plants P1 and P2 cannot ship to warehouse 3. We can also see that warehouse W1 cannot ship to retailers R5, R6 or R7. This means that the demand from retailers R5, R6 and R7 must be met by supply strictly coming from warehouse W3 (with the elimination of W2). So, since only ports P3 and P4 can ship to W3, we can add up their supply and see if it is enough to offset the demand for ports R5, R6 and R7.

The results are:

Supply from P3 + P4: 400

Demand from R5, R6, R7: 450

Since the demand is greater than supply without W2, there is no feasible model to ship all refrigerators using only warehouse 1 and 3.

Part C

Yes it is possible, since we have seen the demand above is 450, and the supply is 400 (without W2). So if you add back in W2, even capping it at 100, it is feasible to meet demand. To solve this we added a line to the Lindo input supply constraints, as follows:

P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 <=100

Basically just adding up all possible routes through W2, and capping them at 100 refrigerators. The optimal was found at:

VARIABLE VALUE REDUCED COST

P1W1R1 50.000000 0.000000 (Ship 50 refrigerators from P1 to R1 through W1)

P1W1R3 100.000000 0.000000 (Ship 100 refrigerators from P1 to R3 through W1)

P2W1R1 50.000000 0.000000 (Ship 50 refrigerators from P2 to R1 through W1)

P2W1R2 150.000000 0.000000 (Ship 150 refrigerators from P2 to R2 through W1)

P2W1R4 150.000000 0.000000 (Ship 150 refrigerators from P2 to R4 through W1)

P2W2R4 50.000000 0.000000 (Ship 50 refrigerators from P2 to R4 through W2)

P2W2R5 50.000000 0.000000 (Ship 50 refrigerators from P2 to R5 through W2)

P3W3R5 100.000000 0.000000 (Ship 100 refrigerators from P3 to R5 through W3)

P3W3R6 150.000000 0.000000 (Ship 150 refrigerators from P3 to R6 through W3)

P4W3R5 50.000000 0.000000 (Ship 50 refrigerators from P4 to R5 through W3)

P4W3R7 100.000000 0.000000 (Ship 100 refrigerators from P4 to R7 through W3)

And the total cost comes to $18,300.00.

Part D

Cijk = Unit transportation cost from source i, through warehouse j, to destination k

ai = amount of supply at source i

bk = amount of demand at retailer k

Xik = number of refrigerators distributed from origin i to retailer k

Minimize z =

Subject to:

Xik >= 0

**Problem 2:** A mixture problem

**Problem 3:** Solving shortest path problems using linear programming.

Part A)

|  |  |
| --- | --- |
| VARIABLE | VALUE |
| B | 2 |
| C | 3 |
| D | 8 |
| E | 9 |
| F | 6 |
| G | 8 |
| H | 9 |
| I | 8 |
| J | 10 |
| K | 12 |
| L | 15 |
| M | 17 |
| A | 0 |

The Lindo input was as follows:

max b + c + d + e + f + g + h + i + j + k + l + m

ST

a = 0

b - a <= 2

c - a <= 3

d - a <= 8

h - a <= 9

a - b <= 4

c - b <= 5

e - b <= 7

f - b <= 4

d - c <= 10

b - c <= 5

g - c <= 9

i - c <= 11

f - c <= 4

a - d <= 8

g - d <= 2

j - d <= 5

f - d <= 5

f - d <= 1

h - e <= 5

c - e <= 4

i - e <= 10

i - f <= 2

g - f <= 2

d - g <= 2

j - g <= 8

k - g <= 12

i - h <= 5

k - h <= 10

a - i <= 20

k - i <= 6

j - i <= 2

m - i <= 12

i - j <= 2

k - i <= 4

l - j <= 5

h - k <= 10

m - k <= 10

m - l <= 2

END

Part B

Adding in vertex Z with no path to vertex a resulted in Lindo throwing an error. It said unbounded solution.

Here is the input used:

NBOUNDED VARIABLES ARE:

SLK 41

Z

OBJECTIVE FUNCTION VALUE

1) 0.9999990E+08

And here is the result:

max b + c + d + e + f + g + h + i + j + k + l + m

ST

a = 0

b - a <= 2

c - a <= 3

d - a <= 8

h - a <= 9

a - b <= 4

c - b <= 5

e - b <= 7

f - b <= 4

d - c <= 10

b - c <= 5

g - c <= 9

i - c <= 11

f - c <= 4

a - d <= 8

g - d <= 2

j - d <= 5

f - d <= 5

f - d <= 1

h - e <= 5

c - e <= 4

i - e <= 10

i - f <= 2

g - f <= 2

d - g <= 2

j - g <= 8

k - g <= 12

i - h <= 5

k - h <= 10

a - i <= 20

k - i <= 6

j - i <= 2

m - i <= 12

i - j <= 2

k - i <= 4

l - j <= 5

h - k <= 10

m - k <= 10

m - l <= 2

m - z <= 5

END

Part C

|  |  |
| --- | --- |
| Vertex to M | Shortest Path |
| A | 17 |
| B | 15 |
| C | 15 |
| D | 12 |
| E | 19 |
| F | 11 |
| G | 14 |
| H | 14 |
| I | 9 |
| J | 7 |
| K | 10 |
| L | 2 |
| M | 0 |

Lindo input:

MAX a + b + c + d + e + f + g + h + i + j + k + l

ST

m = 0

a - b <= 2

a - c <= 3

a - d <= 8

a - h <= 9

b - a <= 4

b - c <= 5

b - e <= 7

b - f <= 4

c - d <= 10

c - b <= 5

c - g <= 9

c - i <= 11

c - f <= 4

d - a <= 8

d - g <=2

d - j <= 5

d - f <= 1

e - h <= 5

e - c <= 4

e - i <= 10

f - i <= 2

f - g <= 2

g - d <= 2

g - j <= 8

g - k <= 12

h - i <= 5

h - k <= 10

i - a <= 20

i - k <= 6

i - j <= 2

i - m <= 12

j - i <= 2

j - k <= 4

j - l <= 5

k - h <= 10

k - m <= 10

l - m <= 2

END

Part D